A pricing mechanism for Femto Base Stations*

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We denote by $\mathcal{Q}$ the reference area of size $Q\ m^2$. Our problem is to fully cover the area and to provide certain level of QoS for users. There are two types of Base Stations (BS) that can be used for this purpose. Macro-BSs that have large transmission ranges but their energy consumption is high, and Femto-BSs that have smaller transmission ranges and relatively smaller energy consumption. Telecoms service providers have full control over functionality of Macro-BSs, while Femto-BSs are acting more phazy and are controlled by home service providers. Hence, from the point of view of a service provider, Femto-BSs are unreliable. In practice, the full coverage would be provided by reliable Macro-BSs and Femto-BSs would be mainly used to enhance the level of QoS in the network.

As discussed above, telecoms service providers have not control over functionality of Femto-BSs. However, they can motivate home service providers to cooperate with them (e.g. activate their Femto-BSs) by porpoising a reasonable price for service they provide. A home service provider can then evaluate its utility according to the price proposed by the telecoms service provider and decide to cooperate if it gets a positive utility. In this draft, we propose a pricing mechanism and evaluate the behavior of home service providers (i.e., the number of active Femto-BSs) for a given price. We mainly focus our attention on Nash equilibrium analysis.

I. Game description

Let N be the number of Femto-BSs in $\mathcal{Q}$. We consider that Femto-BSs are uniformly and independently distributed in the area. We divide time into slots of size one unit, where each Femto-BS can be active or passive during a time slot. Femto-BS i's decision, $1 \leq i \leq N$, about whether to be active during a time slot is determined by BS i's strategy. We denote by $x_i(t) \in A_i = \{0, 1\}$ node i's strategy during time slot t, where i is active if $x_i(t) = 1$ and passive otherwise. $x_i(t)$ is a function of p, where p is the money paid to an active Femto-BS for serving a request (e.g. a session request). We consider that p does't change over time, i.e. we have a single shot game where the strategy of nodes is fixed. Hence, for the sake of simplicity, we drop t from our notation.

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Let r be the transmission range of a Femto-BS. Following policy is used to assign '*active*' Macro and Femto -BSs to arrived requests. When a request arrive, it randomly selects one of the active Femto-BSs that can serve its request (the request must be in transmission range of a Femto-BS to be served by the Femto-BS). If there is no such a Femto-BS, then the request would be served by one of the available active Macro-BS. We consider that requests arrive according to a Poisson distribution with mean λ requests per unit of time, where the position of arrivals are uniformly and independently distributed in $\mathcal{Q}$. We denote by σ the average size of requests.

We consider a linear model for the cost of energy consumed by an active Femto-BS during a time slot:

$$c_{total} = c + ncs, \tag{1}$$

where c_{total} is the cost of total energy consumed by the Femto-BS, $c > 0$ is the cost of energy consumed by an active Femto-BS when its traffic load is zero, $c_s > 0$ is the marginal cost (e.g. energy cost) of serving a request, and n is the total number of the requests served by the Femto-cell. We consider that $p > c_s$, as there is no incentive for Femto-BSs to serve a request if $p \leq c_s$. We consider that the energy consumed by a passive Femto-BS is negligible and hence its cost.

Let $s(h)$ be the '*active*' Femto-BS chosen to serve the h-th request arrived during a time slot ($s(h) = 0$ if the request is served by a Macro-BS). Then, Femto-BS i, $1 \leq i \leq N$, gets utility $p - c_s$ if $i = s(h)$ and 0 otherwise, i.e.,

$$u_i(x \in A, h) = \begin{cases} p - c_s & if\ i = s(h) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $A = A_1 \times A_2 \times \cdots \times A_N$ denotes the set of strategy profiles of Femto-BSs. We denote by $u_i(x) = \sum_{h=1}^{K} u_i(x,h) - cx_i$ the total utility received by Femto-BS i during the time slot, where K is the total number of arrivals in the area. Note that if $x_i = 0$ then $u_i(x,h) = 0$ for any $1 \leq h \leq K$ and hence $u_i(x) = 0$. $u_i(x) = Kq_i(p - c_s) - c$ if $x_i = 1$, where q_i is the fraction of the arrived requests in the area assigned to Femto-BS i. K and q_i are independent random variables, hence the expected utility per time slot received by Femto-BS i is

$$U_i(x) = \begin{cases} \lambda \bar{q}_i (p - c_s) - c & if\ x_i = 1 \\ 0 & otherwise \end{cases} \tag{3}$$

where λ and $\bar{q}_i$ are expected values of K and q_i, respectively.

We denote by $G(U, A)$ the game described above, where $U(x) = (U_1(x), U_2(x), \cdots, U_N(x))$ is the payoff function for any $x \in A$. Our goal is to find Nash equilibrium of $G(U, A)$ for a given price p. In the next section, we consider a specific scenario that r is enough large such that $\mathcal{Q}$ is fully covered by each of the Femto-BSs. We then extend our results to more general case where each Femto-BSs covers only a subset of the area.

II. A SIMPLE SCENARIO: A CLIQUE

We first consider that r is large such that all the points in $\mathcal{Q}$ are covered by a Femto-BS. Let $N_A \leq N$ be the number of active Femto-BSs in the area, i.e. $N_A = |x|$ where $x \in A$ is the strategy played by the Femto-BSs. An arrived request could be served by any of these active Femto-BSs (the request randomly selects one of them). Hence $\bar{q}_i = \frac{1}{N_A}$ is the expected fraction of arrivals in the area assigned to an active Femto-BS when $N_A \geq 1$. Then, (3) can be simplified to

$$U_i(x) = \begin{cases} \frac{\lambda}{N_A}(p - c_s) - c & if\ x_i = 1 \\ 0 & otherwise \end{cases} \tag{4}$$

Now let

$$N_A^c = \min\left(\max\left(0, \left\lfloor \frac{\lambda(p - c_s)}{c} \right\rfloor\right), N\right) \tag{5}$$

where $\left\lfloor \frac{\lambda(p-c_s)}{c} \right\rfloor$ is the nearest integers less than or equal to $\frac{\lambda(p-c_s)}{c}$. N_A^c is the greatest value of $N_A \leq N$ such that $U_i \geq 0$ if $A_i = 1$, for all $1 \leq i \leq N$.

Theorem 1 *$x^* = (x_1^*, x_2^*, \cdots, x_N^*) \in A$ such that $|x^*| = \sum_{i=1}^{N} x_i^* = N_A^c$ is a Nash equilibrium of $G(U, A)$ for the scenario described in this section.*

Proof: See Appendix . ■

Note that the set of Nash equilibriums given by this theorem might not be the set of all possible Nash equilibriums of the game (see following theorem).

Theorem 2 *If $N_A^c > 0$ and if $\frac{\lambda}{N_A^c}(p - c_s) - c = 0$, then $x^* \in A$ such that $|x^*| = N_A^c - 1$ is a Nash equilibrium of $G(U, A)$ for the described scenario.*

Proof: The proof is given in Appendix B ■

Let $\mathcal{E}_1$, $\mathcal{E}_2 \subset \mathcal{A}$ be respectively the set of Nash equilibriums given by Theorems 1 and 2. Note that $\mathcal{E}_2 = \Phi$ if $\frac{\lambda}{N_A^c}(p - c_s) - c \neq 0$ or if $N_A^c = 0$.

Theorem 3 *$\mathcal{E} = \mathcal{E}_1 \cup \mathcal{E}_2$ contains all possible Nash equilibriums of $G(U, A)$ for the scenario described in this section.*

Proof: The proof is given in Appendix C. ■

III. A NETWORK SETTING SCENARIO

In this section, we consider a more general scenario where r is small and hence a Femto-BS covers only a subset of points in $\mathcal{Q}$. As described in Section I, we consider that Femto-BS are uniformly and independently distributed in the area. Similar to previous section, we use N_A to denote the number of active Femto-BSs in the area. Hence, $\frac{N_A}{Q}$ is the density of active Femto-BSs per unit of area.

We define the 'area coverage' (f_a) as the fraction of the geographical area covered by one or more Femto-BS. It is shown in [1] that if the area is enough large, then

$$f_a = 1 - e^{-\frac{N_A}{Q}\pi r^2}. \tag{6}$$

Recall that based on our assumptions, the position of Femto-BSs and arrivals are uniformly and independently distributed in $\mathcal{Q}$, and a user randomly selects its server from the set of active Femto-BSs that can serve its request. Hence, the expected fraction of arrivals in the area assigned to an active Femto-BS is $\bar{q}_i = \frac{1-e^{-N_A\pi r^2/Q}}{N_A}$. Using this results, (3) is simplified to

$$U_i(x) = \begin{cases} \frac{\lambda(1-e^{-N_A\pi r^2/Q})}{N_A}(p-c_s) - c & if\ x_i = 1 \\ 0 & otherwise \end{cases} \tag{7}$$

First we prove following lemma.

Lemma 1 $g(N_A) = \frac{\lambda(1-e^{-N_A\pi r^2/Q})}{N_A}(p-c_s) - c$ *is a decreasing function in* $N_A > 0$.

Proof: See Appendix D. ■

We now define $N_A^n = \min(\max(0, \lfloor y \rfloor), N)$, where $\frac{\lambda(1-e^{-y\pi r^2/Q})}{y}(p-c_s) - c = 0$. N_A^n is the greatest value of $N_A < N$ such that $\frac{\lambda(1-e^{-N_A\pi r^2/Q})}{N_A}(p-c_s) - c \geq 0$. From Lemma 1 and following the same argument used to prove Theorems 1, 2, and 3 we can prove following results.

Theorem 4 *In a network setting, as described above,* $x^* \in A$ *such that* $|x^*| = N_A^n$ *is a Nash equilibrium of* $G(U, A)$.

Theorem 5 *In the scenario described in this section,* $x^* \in A$ *such that* $|x^*| = N_A^n - 1$ *is a Nash equilibrium of* $G(U, A)$ *if* $N_A^n > 0$ *and if* $\frac{\lambda(1-e^{-N_A^n\pi r^2/Q})}{N_A^n}(p-c_s) - c = 0$.

Theorem 6 *Let* $\mathcal{E}^n \subset \mathcal{A}$ *be the set of Nash equilibriums given by Theorems 4 and 5,* $\mathcal{E}^n$ *contains all possible Nash equilibriums of the game for the scenario described in this section.*

REFERENCES

[1] B. Liu and D. Towsley. A study on the coverage of large-scale sensor networks. In *Proc. of MASS*, 2004.

APPENDIX

A. Appendix A

We need to show that $\forall i, x_i \in A_i = \{0,1\},\ x_i \neq x_i^* :\ U_i(x_i^*, x_{-i}^*) \geq U_i(x_i, x_{-i}^*)$, where x_{-i}^* is the strategy of all players except i. Note from (4) that $U_i(x_i^*, x_{-i}^*) = 0$ (resp. $U_i(x_i, x_{-i}^*) = 0$) if $x_i^* = 0$ (resp. $x_i = 0$).

We first consider the case that $N_A^c = 0$, i.e. $\frac{\lambda(p-c_s)}{c} < 1$. In this case $x^* = (0, 0, \cdots, 0)$ is a Nash equilibrium as $U_i(x_i^*, x_{-i}^*) = 0$ and $U_i(x_i = 1, x_{-i}^*) = \lambda(p - c_s) - c < 0$, i.e. $U_i(x_i^*, x_{-i}^*) > U_i(x_i = 1, x_{-i}^*)$, for any $1 \leq i \leq N$.

We then consider the case that $1 \leq N_A^c \leq N$. For any $1 \leq i \leq N$:

- If $x_i^* = 0$, then $U_i(x_i^*, x_{-i}^*) = 0$. Moreover, it is easy to show that $\frac{\lambda}{N_A}(p - c_s) - c$ is a decreasing function of $N_A > 0$. Hence, from the definition of N_A^c we have $U_i(x_i = 1, x_{-i}^*) = \frac{\lambda}{N_A^c+1}(p - c_s) - c \leq 0$. Thus, the utility of i would not increase if it switches to active mode.
- If $x_i^* = 1$, then $U_i(x_i^*, x_{-i}^*) = \frac{\lambda}{N_A^c}(p - c_s) - c \geq 0$ (this comes from the definition of N_A^c) and $U_i(x_i = 0, x_{-i}^*) = 0$. Hence i would not get greater utility if it switches to passive mode.

The proof is completed.

B. Appendix B

The proof is similar to the proof of Theorem 1. For any $1 \leq i \leq N$, let $x_i \neq x_i^* \in \{0,1\}$ and x_{-i}^* be the strategy of all players except i.

We first consider the case that $N_A^c = 1$. In this case $x^* = (0, 0, \cdots, 0)$ is the Nash equilibrium as $U_i(x_i^*, x_{-i}^*) = 0$ and $U_i(x_i = 1, x_{-i}^*) = \lambda(p - c_s) - c = 0$, i.e. $U_i(x_i^*, x_{-i}^*) = U_i(x_i = 1, x_{-i}^*)$, for any $1 \leq i \leq N$.

We then consider the case that $2 \leq N_A^c \leq N$. For any $1 \leq i \leq N$:

- If $x_i^* = 0$, then $U_i(x_i^*, x_{-i}^*) = 0$ and $U_i(x_i = 1, x_{-i}^*) = \frac{\lambda}{N_A^c}(p - c_s) - c = 0$. Thus i would not get greater utility if it switches to active mode.
- If $x_i^* = 1$, then $U_i(x_i^*, x_{-i}^*) = \frac{\lambda}{N_A^c-1}(p - c_s) - c \geq 0$ (this comes from the fact that $\frac{\lambda}{N_A^c}(p - c_s) - c$ is a decreasing function of $N_A^c > 0$) and $U_i(x_i = 0, x_{-i}^*) = 0$. Hence, the utility of i would not increase if it switches to passive mode.

The proof is completed.

C. Appendix C

Let $x^{'*} \notin \mathcal{E}$, $x^{'*} \in A$, be a Nash equilibrium of the game and let $\left|x^{'*}\right| = N^{'}$, then for all $1 \leq i \leq N$

$$U_i(x^{'*}) = \begin{cases} \frac{\lambda}{N_A^{'}}(p - c_s) - c & if\ x_i^{'*} = 1 \\ 0 & otherwise \end{cases}$$

For any $1 \leq i \leq N$, let $x_i \neq x_i^{'*} \in \{0, 1\}$ and $x_{-i}^{'*}$ be the strategy of all players except i.

We first consider the case that $N_A^{'} = 0$, i.e. $x^{'*} = (0, 0, \cdots, 0)$. In this case $U_i(x_i^{'*}, x_{-i}^{'*}) = 0$ for all $1 \leq i \leq N$. As $x^{'*}$ is a Nash equilibrium of the game, then $U_i(x_i = 1, x_{-i}^{'*}) = \lambda(p - c_s) - c \leq 0$. Using (5), we have $N_A^c = 0$ and hence $(0, 0, \cdots, 0) \in \mathcal{E}_1$ (see Theorem 1).

Now consider the case that $1 < N_A^{'} < N$. Note that as $1 < N_A^{'} < N$, there is at least one active and at least one passive Femto-BSs in the area. Suppose $1 \leq i \leq N$. From the definition of the Nash equilibrium, we know that if $x_i^{'*} = 0$, then $U_i(x_i^{'*}, x_{-i}^{'*}) \geq U_i(x_i = 1, x_{-i}^{'*})$, i.e. $\frac{\lambda}{N_A^{'}+1}(p - c_s) - c \leq 0$. If $x_i^{'*} = 1$, then $U_i(x_i^{'*}, x_{-i}^{'*}) \geq U_i(x_i = 0, x_{-i}^{'*})$ and hence $\frac{\lambda}{N_A^{'}}(p - c_s) - c \geq 0$.

- If $\frac{\lambda}{N_A^{'}+1}(p - c_s) - c = 0$, then $N_A^c = N_A^{'} + 1$ using (5). Applying Theorem 2, we can easily show that $x^{'*} \in \mathcal{E}_2$.
- If $\frac{\lambda}{N_A^{'}+1}(p - c_s) - c < 0$, then $N_A^c = N_A^{'}$. It follows from the fact that $\frac{\lambda}{N_A^{'}}(p - c_s) - c \geq 0$ and is a decreasing function of $N_A^{'}$. Then using Theorem 1, we have $x^{'*} \in \mathcal{E}_1$.

Finally, we consider the case that $N_A^{'} = N$. As $x^{'*} = (1, 1, \cdots, 1)$ is a Nash equilibrium of the game, then $U_i(x_i^{'*}, x_{-i}^{'*}) \geq U_i(x_i = 0, x_{-i}^{'*})$, i.e. $\frac{\lambda}{N}(p - c_s) - c \geq 0$. Using (5), we have $N_A^c = N$ and from Theorem 1 we can show that $x^{'*} \in \mathcal{E}_1$.

The proof is completed.

D. Appendix D

We show that derivative of $g(N_A)$ with respect to N_A is negative for any $N_A > 0$

$$\begin{aligned}
\frac{d}{dN_A} g(N_A) &= \lambda(p - c_s) \frac{(N_A \pi r^2/Q) e^{-N_A \pi r^2/Q} - (1 - e^{-N_A \pi r^2/Q})}{N_A^2} \\
&= \lambda(p - c_s) \frac{(1 + N_A \pi r^2/Q) e^{-N_A \pi r^2/Q} - 1}{N_A^2} \\
&= \lambda(p - c_s) \frac{(1 + N_A \pi r^2/Q) - e^{N_A \pi r^2/Q}}{e^{N_A \pi r^2/Q} N_A^2} \\
&= \lambda(p - c_s) \frac{(1 + N_A \pi r^2/Q) - (1 + N_A \pi r^2/Q + (N_A \pi r^2/Q)^2/2 + \cdots)}{e^{N_A \pi r^2/Q} N_A^2} \\
&= \lambda(p - c_s) \frac{(-(N_A \pi r^2/Q)^2/2 - (N_A \pi r^2/Q)^3/6 - \cdots)}{e^{N_A \pi r^2/Q} N_A^2} \\
&\leq 0
\end{aligned}$$

where the before last equality follows from the Taylor expansion of $e^{N_A \pi r^2/Q}$, and inequality is true if $N_A > 0$. The proof is completed.